Propagation of electromagnetic waves

Energy per volume dV in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dV}=u_\lambda d\lambda$$; u_λ = specific energy density.

Example: black-body radiation $\frac{dE}{dV}=8\pi h\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1}=\frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$

Total (integral over all wave lengths): $4\sigma/c\, T^4$

Power emitted per area dA into solid angle $d\Omega$ in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dt\,dA\,d\Omega}=\cos\theta\cdot I_\lambda(\theta,\varphi)\,d\lambda$$; I_λ = specific intensity.

Average: $\langle I_\lambda\rangle=\frac{1}{4\pi}\iint I_\lambda(\theta,\varphi)\,d\Omega=\frac{c}{4\pi}u_\lambda$. Ex.: black-body: $\langle I_\lambda\rangle=\frac{2hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$

Power emitted in positive (neg.) z-direction per area dA (perpendicular to z) and per $d\lambda$:

Radiation flux density $F_\lambda=\int_0^{2\pi}d\varphi\int_{0(\pi/2)}^{\pi/2(\pi)}\cos\theta\cdot I_\lambda(\theta,\varphi)\sin\theta\,d\theta$

For isotropic specific intensity (for top hemisphere): $\Rightarrow F_\lambda=\pi\langle I_\lambda\rangle$

Black-body radiation: $F_\lambda d\lambda=\frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}=2\pi hc\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1}$

Radiation pressure in z-direction: $dP_\lambda^z=\frac{2}{c}d\lambda\int_0^{2\pi}d\varphi\int_{0(\pi/2)}^{\pi/2(\pi)}\cos^2\theta\cdot I_\lambda(\theta,\varphi)\sin\theta\,d\theta$

For isotropic specific intensity (for top hemisphere): $\Rightarrow dP_\lambda^z=\frac{4\pi}{3c}\langle I_\lambda\rangle d\lambda=\frac{1}{3}u_\lambda d\lambda$

Further properties of Black-body radiation

Maximum intensity per unit wave length interval for λ = 2.9 mm / T [K]

(Wien displacement law)

Total power emitted (intensity integrated over all wave lengths) $I=\sigma T^4$

(Stefan-Boltzmann equation)

Total luminosity of spherical black-body of radius R: $L=4\pi R^2\sigma T^4$

Apparent brightness (flux density) at distance d: $F=\frac{L}{4\pi d^2}$