PHYSICS 313 - Winter/Spring Semester 2017 - ODU

Astrophysics - Problem Set 9 – Solution

Problem 1

Mark each of the following items with "Y" or "T" if they can be found in our own galaxy, the Milky Way, and with "F" or "N" if they don't exist within the Milky Way (including its halo):

1a) Individual (unbound, single) quarks. **N**
1b) Positrons. **Y**
1c) Neutrinos. **Y**
1d) Photons. **Y**
1e) Some type of dark matter. **Y**
1f) Hydrogen gas **Y**
1g) Dust particles. **Y**
1h) 10 Trillion main sequence stars. **N** ["only" 20 billion]
1i) White dwarfs. **Y**
1j) More than 1000 planets. **Y**
1k) Neutron stars. **Y**
1l) Sun-mass black holes. **Y**
1m) (At least one) supermassive black hole. **Y**
1n) Quasars. **N**
1o) Globular clusters. **Y**
1p) Magnetic fields. **Y**
1q) Spiral arms. **Y**
1r) Central bulge. **Y**
1s) Noodles. **Y** [This may seem silly, but apart from real pasta - which our Galaxy certainly contains - new evidence points to so-called "plasma noodles" spanning many light-years; not to mention the "nuclear pasta" in the outer parts of neutron stars…]

Problem 2

The following is a set of multiple choice questions. Answer each with a single digit:

2a) Which of the following statements about the Milky way is correct? **2**
1 – Stars in the spiral arms collide with each other all the time.
2 – Most stars in the disk are moving roughly with the same speed.
3 – A star that is born in a spiral arm stays within that same arm for all of its life.
4 – Our galaxy contains no black holes with more than 10 solar masses.

2b) Which of the following types of electromagnetic radiation is **un**suitable to observe the center of our galaxy? **3**
1 – Infrared radiation
2 – Radio waves
3 – Visible light
4 –x-rays.

Problem 3 – Show your work (not only final results)

Some halo stars and globular clusters circle our Milky Way at radii as large as 50 kpc. Assume they move with tangential velocity 200 km/s. Calculate the total mass required for the Milky Way to explain this motion in terms of Keplerian orbits (i.e., assuming a spherical mass *M* inside the orbit, calculate the size of *M* in units of solar masses, $2\text{x}10^{30}$ kg). Compare this number with the visible luminosity, which is roughly 20 billion times that of the sun.
Note: 1 pc = $3\text{x}10^{16}$ m.

Answ.: Setting the centripetal acceleration, $a = v^2/r$, equal to the gravitational acceleration due to a central mass *M* yields

$$\frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow \quad M = \frac{rv^2}{G} = \frac{(50{,}000 \times 3\cdot 10^{16}\,m)(200{,}000\,m/s)^2}{6.67\cdot 10^{-11}} = 9\cdot 10^{41}\,kg = 4.5\cdot 10^{11}\,M_{sun}$$

So, compared to the sun, the whole Milky Way appears to be more than 20 times as massive per unit of luminosity (making it likely that it contains "dark matter").

Problem 4 – Final numerical results suffice, but you may show your work "in case"

Calculate the Schwarzschild radius of a black hole with mass $4\cdot 10^6$ M_{sun}. Compare to the following sizes:
1) The radius of a star with the same mass and the density of a typical red giant like Betelgeuse (10^{-5} kg/m^3; 1/100 millionth the density of water!)
2) The diameter of an object that varies on the time scale of 1 hour (assuming that for causality to hold, the object cannot be larger than the distance traveled by light in 1 hour)
3) The closest approach of the orbit of star S0-16, which has an orbit with major half axis of 900 AU and an eccentricity of e = 0.95.
4) The resolution of a long-baseline radio interferometer, 10^{-3} arcsec, over the distance from Earth to the galactic center (8 kpc).

Answ.: Using $r_S = 2GM/c^2 = 3\text{km}\ M/M_{sun}$, we get a Schwarzschild radius of $1.2\cdot 10^{10}$ m = 0.08 AU, well inside the orbit of Mercury. In comparison,
1) A star with mass $4\cdot 10^6$ M_{sun} and density 10^{-5} kg/m^3 would have a volume of $8\cdot 10^{41}$ m^3 and therefore (assuming a sphere) a radius of $r = 5.75\cdot 10^{13}$ m = 384 AU, which is much larger than the observed upper limits (below).
2) Light travels 300,000 km in 1 second, which comes out to 7.2 AU in one hour. This would be the diameter of an object that fluctuates coherently within 1 hour. The radius would be even smaller, 3.6 AU.
3) This distance of closest approach for S0-16 would be $(1-\varepsilon)\cdot 900$ AU = 45 AU, which is larger than the upper limit in 2) and definitely can accommodate the calculated Schwarzschild radius, but is incompatible with a "Betelgeuse"-type giant star.

4) The resolution of such an interferometer would be 8000 pc times 10^{-3} arcsec. By the very definition of a parsec, 1 pc times a resolution of 1 arcsec equals 1 AU. Therefore, 8000 pc times 10^{-3} arcsec must equal 8 AU, again in agreement with 2) and other limits as well as the Schwarzschild radius, but inconsistent with a stellar object. So the best bet is indeed a Black Hole.