PHYSICS 313 - Winter/Spring Semester 2017 - ODU

Astrophysics - Problem Set 6 – Solution

Problem 1

Please answer the following questions with "Y" or "N":

1a) Do we need Quantum Mechanics to truly understand the final stages of a star's life? **Y**
[Degeneracy pressure is a direct consequence of the Pauli exclusion principle and Heisenberg's uncertainty relationship, both of which are pillars of Quantum Mechanics.]

1b) Do we need Special Relativity to understand the stability limit of white dwarfs? **Y**
[The Chandrasekar limit comes about because has electrons gain too high momenta, they become relativistic – i.e., their speeds are close to c – which means their pressure is not increasing as quickly as necessary to counteract gravity].

1c) Do we need General Relativity to understand the final fate of the most massive stars? **Y**
[The most massive stars end up as black holes which are only described properly by GR]

1d) Do we need Nuclear Physics to understand the life cycle of stars? **Y**
[Of course – all the various nuclear reactions going on starting near the ZAMS line until the very end…]

1e) Do we need Particle Physics to understand supernovae? **Y / N**
[If by particle physics we understand, among other things, the interactions between neutrinos and other types of matter. But if you chose "N", I'll let that stand – so it's a freebie!]

1f) Do we need all of the above to understand proto-stellar clouds? **N**
[Protostellar clouds are not very dense, quite cold and don't involve nuclear reactions. They are way too dilute to require General Relativity, and the speeds involved are a lot smaller than c.]

Problem 2

The following is a set of multiple choice questions. Answer each with one single digit:

2a) Which of the following objects is **not** stabilized through degeneracy/Fermi pressure (a.k.a. the Pauli principle)? **1**

1 – Zero-age main sequence stars [They are stabilized through ordinary gas pressure coming from high temperatures inside, which in turn are maintained via nuclear fusion]
2 – Carbon/Oxygen cores of giant stars [That's where degeneracy pressure starts to play a role]
3 – Neutron stars
4 – White dwarfs

2b) Why are white dwarfs beyond a certain mass ("Chandrasekar limit") unstable? **3**

1 – Because nuclear fusion stops when the mass is too high.
2 – Because degeneracy pressure becomes negligibly small at high density
3 – Because electrons become highly relativistic at large Fermi momenta (high density)
4 – Because they emit gravitational waves if they get too massive

Problem 3

(Numerical result only): Calculate the radius of a white dwarf with exactly one solar mass using a simple Fermi-gas model (ignoring relativity). Repeat the calculation with 1.2 times solar mass and show that the new radius is smaller (by how much?).

Answ.: plugging in all the numbers gives **7233 km** for the first case and **6807 km** for the second case (6% smaller).

Note: In equilibrium, we have $R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e G M^2}\left(\frac{9\pi}{4}\right)^{2/3}$

with $p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3}; \quad n = \frac{N_{tot}}{V}; \quad N_{tot} = \frac{M_{star}}{1\text{ g}}\frac{N_A}{2}$

Problem 4

(Text only): In your own words, describe how and where the most heavy elements (beyond iron, all the way to uranium) have been (and still are being) produced. 3-4 sentences

Answ.: Elements up to iron are produced in very heavy stars, in their cores, close to the end of their lifespan. Once all nuclear material has fused into iron, further fusion does not liberate energy, but actually requires energy input, leading to the core collapse of these supergiants. While the nuclei in the core are largely destroyed by this process (leading to a neutron star), there are many processes including rapid and slow neutron capture processes in the outer layers of the supernova that can produce heavier elements. As neutrons are added in rapid succession, these heavy nuclei undergo beta-decay (emitting electons and anti-neutrinos) to convert some of the excess neutrons into protons, yielding nuclei with higher Z all the way up to Uranium (Z = 92).