PHYSICS 313 - Winter/Spring Semester 2016 - ODU

Astrophysics - Problem Set 3 – Solution

Problem 1

Please answer the following questions with "Y" or "N":

1a) Do we need Special Relativity to understand the Doppler shift of light emitted from fast-moving objects? **Y**

1b) If a star would pass our solar system with very high speed, there would be a Doppler shift even when it moves exactly perpendicular to our line of sight. True? **Y** [This is due to the time dilation, which means from our own vantage point the "clock" that determines the period and thus the frequency of the emitted light in the moving system "runs slow"].

1c) The absorption spectrum of a star is completely determined by its chemical composition alone. True? **N** [The temperature plays also an important role]

1d) Because hydrogen is the most common element in stars, the Hydrogen Balmer absorption lines are the most prominent (dark) ones for all stars. True? **N** [No, because to absorb one of these photons, a hydrogen atom would have to be in its excited state, which only a tiny fraction will be in an ordinary star's photosphere]

1e) Blue stars tend to be brighter than red stars. True? **Y** [Stefan-Boltzmann law and Wien's displacement law]

1f) A red star can never be more luminous than any blue one. True? **N** [It could have a higher luminosity if it has a **much** larger surface area, as for instance red supergiants do.]

Problem 2

The following is a set of multiple choice questions. Answer each with single digit:

2a) Which one of the following properties does not tend to **increase** with temperature of a main sequence star? **1** [All others do tend to increase with temperature - see slides about the Hertzsprung-Russell diagram]

1 – Its distance from Earth
2 – Its size
3 – Its mass
4 – Its total luminosity
5 – Its emission of blue light

2b) Which of the following types of stars is on the "main sequence" in the Hertzsprung-Russel diagram? **1**

1 – M7 spectral type stars
2 – Blue Supergiants
3 – White dwarfs
4 – Neutron Stars
5 – Red Giants

2c) Which of the following statements about the Special Theory of Relativity is true? **3**

1 – Two events that are simultaneous in one inertial system must be simultaneous in any other one.
2 – Light emitted from the head of the locomotive of an onrushing train going 90% of the

speed of light will appear to move towards me at 1.9 c. [No, the speed of light is the same in all coordinate systems].

3 – I observe the clock in a system moving at 60% of c relative to me. When my own clock tells me 10 seconds have elapsed, that other clock shows only 8 elapsed seconds. [True: the "moving clock" runs slow, so the ratio of time elapsed in S to that in S' is given by $\gamma = \frac{1}{\sqrt{1-0.6^2}} = \frac{10}{8}$].

4 – The length of a rigid object is the same as measured from an inertial system in which the object is at rest vs. from an inertial system moving at high speed relative to the first one. [No, because of length contraction.]

Problem 3

The escape velocity of Earth is 11.2 km/s (i.e., an object that reaches that velocity can leave Earth's gravitational field for good). Assuming the mass of a hydrogen atom as 1.66 x10^{-27} kg, calculate the fraction of all hydrogen atoms with enough kinetic energy to reach this escape velocity, using the (simplified) probability factor $\frac{1}{e^{E_{kin}/kT}}$. Take the temperature at the surface of Earth as 300 K. (You may ignore any "degeneracy" and "Normalization" factors here, although in practice they play a big role).

ANSW.: We use the kinetic energy $E_{kin} = \frac{m}{2}v^2 = 1\cdot 10^{-19}$ J of a hydrogen atom at escape velocity and plug in. The result is $1.2 \cdot 10^{-11}$. This seems like a very small probability, but over the age of Earth nearly every single (free) hydrogen atom in the atmosphere has disappeared this way.

Problem 4

XC [Text only]: Assume you observe a pair of stars (a binary star system), 1 parsec from Earth, that are rotating around each other in a plane that you are viewing edge-on. With your high-resolution telescope, you find that they appear to fluctuate in apparent distance between zero and 1" (arc-second = 1/3600 of a degree). You observe their red/blueshift as a function of time and you find that when one of them is redshifted by a certain amount, the other one is blueshifted by exactly the same amount. After ½ year, the pattern is exactly opposite – the formerly redshifted one is now blueshifted and vice versa. The whole game repeats itself exactly after 1 year. What can you conclude about the masses of both stars? [You don't need any math, just some clever arguments. Make sure you explain your reasoning.]

Answ.: Since the pair is "only" 1 parsec away, we can measure their absoluted distance using parallax. By definition, 1" at 1 parsec distance corresponds to 1 A.U., i.e. the two stars are exactly as far apart as Earth and Sun. (Their distance APPEARS to fluctuate due to the fact that we view their motion edge-on). Furthermore, since the ABSOLUTE size of their blue/redshifts are the same, it must be true that they both have the same velocities at all times, which means that they are rotating around a common point right in the middle between them. This in turn implies that they must have the exact same mass $m_1/m_2 = v_2/v_1$). Finally, using the equation

$$\omega^2 = \frac{G(M+m)}{a^3} = \frac{G(2m)}{a^3}$$

and the fact that the angular velocity $\omega = 2\pi/T$, we realize that the sum of those two masses must be equal to the mass of the sun, since the distance AND the period T is the same as for planet Earth's travel around the latter. Hence, each of these two stars has ½ the mass of sun.