Astrophysics - Problem Set 2 – Solution

Problem 1

Please answer the following questions with "Y" or "N":
   1a) Do we need Quantum Mechanics to truly understand the light spectra emitted by stars? **Y**
   1b) The Wien displacement law states that there is a cut-off frequency (above which no electromagnetic radiation can be emitted), depending on the temperature of a body. True? **N** [It is a relationship between the **wave length** for which the light spectrum has **maximum intensity** per wave length band and T; in any case, there is no "cutoff frequency" (although there is also a frequency for which the intensity per frequency band is maximum – interestingly it corresponds to a DIFFERENT wavelength…)]
   1c) Two different transitions to the ground state of some atom yield light of 121.6 nm and 102.6 nm wave length. Should there (in principle) also be light emitted by this atom with a wave length of $(1/102.6 – 1/121.6)^{-1}$ nm = 656 nm? **Y** [The first 2 are from the Lyman series of hydrogen, going from the 2nd and 3rd eigenstate to the ground state. The third is from the Balmer series and corresponds to a transition from the 3rd to the 2nd eigenstate.]
   1d) The dark lines observed in the light spectrum emitted from stars are unrelated to the bright lines observed in discharge lamps filled with a gaseous element. True? **N** [They are actually the same – i.e., at the same wave lengths.]
   1e) A perfect blackbody radiator does not have any dark lines in its light spectrum. True? **Y**
   1f) The line spectrum emitted from a discharge lamp is due to the atomic electrons spiraling into the atomic nuclei, continuously losing energy. True? **N** [They do not **continuously** lose energy, instead they give off energy in a simple "quantum jump" going from one eigenstate to another.]

Problem 2

The following is a set of multiple choice questions. Answer each with one single digit:
2a) Which of the following properties determines the light spectrum of a star? **4**
   1 – Its core temperature (at its center) [No, only its surface temperature]
   2 – Its size
   3 – Its mass
   4 – Its surface temperature and chemical composition [because that determines the observed absoption lines.]

2b) Which of the following equations is NOT relevant to the question: "At which wave lengths does a given element (e.g., hydrogen) emit light?" **1**
   $1 – E = mc^2$
   $2 – E = hf.$
   $3 – c = f\lambda$
   $4 – \mathbf{H}\psi = E\psi.$

Problem 3

Assume the sun's photosphere has a temperature of 5800 K. Answer the following questions just with numerical results (including units where appropriate); you may write down intermediate results for partial credit:

3a) What would be the "apparent red magnitude", m_{red}, of sun if watched through a red filter that only allows light with wave lengths between 650 nm and 660 nm to pass?
(*Hint:* Use the ratio of the intensity within this wave length range according to the Planck blackbody radiation formula = second to last equation below, and compare to the total intensity according to the Stephan-Boltzmann law = last equation below. Don't confuse the Stefan-Boltzmann constant σ with the Thompson cross section! Since the brightness *F* as measured from Earth is directly proportional to the intensity *I*, you can simply replace the ratio of brighnesses with the ratio of intensities dI/I to calculate m_{red} from the last equation in the first line below.) $m_{red} = -21.86$

3b) Repeat for the "apparent violet magnitude", with a filter from 420 nm to 430 nm.
$m_{violet} = -21.97$

3c) How would your answer for 3b) change if the sun's temperature were raised to 11,600 K? (*Hint*: Calculate the Planck blackbody radiation intensity within this wave length range due to the new temperature – but to calculate the apparent magnitude, you can still compare to the total luminosity of our "real sun"!) $m_{violet} = -25.2$

3d) By comparing the known luminosity of the sun (see below) to the Stephan-Boltzmann law, derive the radius of the sun. **690,000 km** (true value: 695,800 km).

<u>Explanation</u>: For parts 3a) – 3c) we simply use the blackbody radiation equation (2nd from last below) with $d\lambda$ = 10 nm (length of the indicated range) and λ in the middle of the indicated range (i.e., either 655 nm or 425nm.). The resulting intensity (flux density) is then divided by the total intensity from the Stefan-Boltzmann law (last equation below). The ratio of these two is inserted into the last equation on the first line, with m_2 = red (or violet) magnitude and $m_1 = m_{sun} = -26.74$ corresponding to $I_{tot} = \sigma T^4$. Part 3d) follows directly from $L = 4\pi R^2 \sigma T^4$ and a little algebra.

Note: $L_{sun} = 3.84 \cdot 10^{26}$ W, m_{sun} = -26.74 and $m_2 - m_1 = 2.5 \lg(F_1/F_2)$ with

$$F = \frac{L}{4\pi d^2} , \quad L = 4\pi R^2 \sigma T^4 , \quad dI = \frac{\Delta E}{\Delta area \Delta t}(\lambda \ldots \lambda + d\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1} , \quad I_{tot} = \sigma T^4$$

Problem 4

(Text only): In your own words, describe how scientists first determined that certain bright emission lines in the spectrum of the sun indicated the presence of a **new** element, not yet observed on Earth. Relate your narrative to information covered in class, explaining the reasoning why it had to be a **new** element. Do NOT copy verbatim from the web, each other or a book - use your own words! (2-3 paragraphs)

<u>Ans.:</u> The element helium was first discovered on the sun, not on Earth. This is the reason it was given its name, after the greek word "helios" for the sun (god). The discovery was made by a French astronomer, Pierre-Jules-César Janssen, who observed a strong yellow line in the emission

spectrum of the chromosphere of the sun (the thin "atmosphere" directly above the photosphere). He was able to observe this emission line only because the main (much brighter) light from the sun was obscured by a solar eclipse. It was also observed by other astronomers.

Since no known element (at the time) had an emission line with exactly the same wave length, it was concluded that this must be a new element. The argument rests on the fact that each element has a unique set of emission (and absorption) lines with specific wave lengths, like a fingerprint, with no 2 elements having exactly the same lines. This is due to the fact (unknown at the time) that, according to quantum mechanics, the light emitted or absorbed by atoms is quantized into photons, each with an amount of energy given precisely by its wave length ($E = hc/\lambda$). In turn, quantum mechanics also predicts that the possible energy levels of atoms are quantized, as well, meaning that the allowable photon energies (and hence wave lengths) must precisely match the difference between two of these energy levels.

Later, Helium was also observed on Earth. It is a decay product of superheavy elements like Uranium, Radium etc., who give off helium nuclei – "alpha particles" – during their radioactive decays. Hence, Sir William Ramsay discovered it (27 years after the original observation on the sun) when he studied minerals containing Uranium, and confirmed that it emitted the same wave length of light.