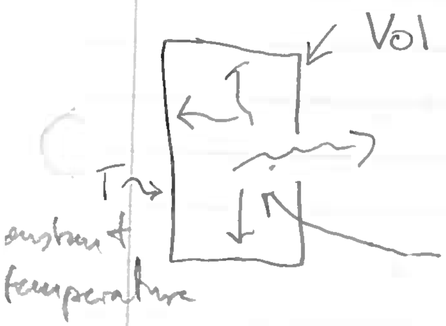
1-21-15

Shaun Sheppard

"Ideal Blackbody"

Absorbs all possible wavelengths → emits all possible w.l.



$$u(f) = \frac{\text{Energy}}{\Delta f \text{ Volume}} \quad \left[\frac{J}{m^3} \Big/ Hz \right]$$

= frequency Interval [Hz]

classical thermodynamics:

$$\frac{\langle E \rangle}{d.o.f} = \frac{1}{2} kT$$

degree of freedom

d.o.f. Electromagnetic waves ?

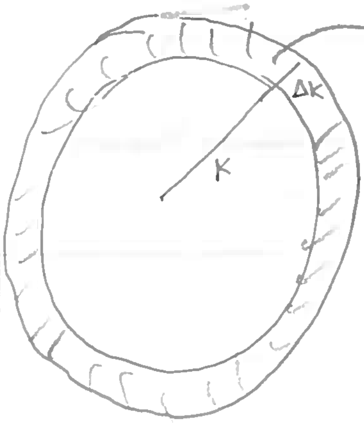
2 · "volume" in $\vec{k}$ space

polarization directions

$$\vec{E}(t, \vec{r}) = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$$

remember: $|\vec{k}| = \frac{2\pi f}{c}$

$|\vec{k}| = k, \ldots, k + \Delta k$



Volume = $\underbrace{\text{surface } 4\pi k^2}\cdot \Delta k$

$\Downarrow$

in "frequency space": $\left(4\pi \left(\frac{f}{c}\right)^2 \cdot \frac{\Delta f}{c}\right) \cdot 2$

$\frac{\text{Energy}}{\Delta f \text{ volume}} \longrightarrow \sim \frac{8\pi f^2}{c^3} \Delta f \left(\cdot \frac{1}{2} kT\right)$ ← classical expectation

→ can't be true! ("ultraviolet catastrophe" → ∞ energy!)

$E_\gamma = hf$

Boltzmann Distribution

$n(E) \sim \frac{1}{e^{E/kT}}$

number of photons / Volume in frequency band $f \ldots f + \Delta f$

$$\frac{n_\gamma}{Vol} = \frac{8\pi \frac{f^2}{c^3} \Delta f}{e^{hf/kT} - 1}$$

because photons are bosons

Plank's constant

$$\frac{\Delta E}{Vol}\left[\frac{J}{m^3}\right] = \frac{8\pi h f^3 / c^3}{e^{hf/kT} - 1} \Delta f$$

since each photon has energy hf

$$f = \frac{c}{\lambda}$$

$$|\Delta f| = \frac{c}{\lambda^2} |\Delta \lambda| \qquad \left(\frac{df}{d\lambda} = -\frac{c}{\lambda^2} \right)$$

$$u(\lambda) = \frac{8\pi h c}{\lambda^5} \frac{\Delta \lambda}{e^{\frac{hc}{\lambda}/kT} - 1}$$

only 'forward' direction counts

$$2\pi \int_0^{\pi/2} \cos(\theta) \sin\theta \, d\theta = \pi$$

vs. $2\pi \int_0^{\pi} \sin\theta \, d\theta = 4\pi$

$\cdot \frac{c}{4}$

$$\boxed{\text{Intensity} = I(\lambda) = \frac{2\pi h c^2}{\lambda^5} \frac{\Delta \lambda}{e^{hc/\lambda kT} - 1}} = \frac{\text{energy radiat}}{\text{unit time} \cdot \text{unit a}}$$

$\left[\frac{W}{m^2}\right]$

$$L = \int_0^\infty I(\lambda)\, d\lambda = \sigma T^4$$

Stefman - boltzmann equation

$$\lambda_{max} \cdot T = 2.9 \text{ mm K}$$