Greek Alphabet

Capital	Α	Β	Γ	Δ	Ε	Ζ	Η	Θ	Ι	Κ	Λ	Μ
Lowercase	α	β	γ	δ	ε	ζ	η	θ, ϑ	ι	κ	λ	μ
Name	alpha	beta	gamma	delta	epsilon	zeta	eta	theta	iota	kappa	lambda	mu
Capital	Ν	Ξ	Ο	Π	Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω
Lowercase	ν	ξ	ο	π	ρ	σ	τ	υ	ϕ, φ	χ	ψ	ω
Name	nu	xi	omicron	pi	rho	sigma	tau	upsilon	phi	chi	psi	omega

Important constants:

Speed of light: $c = 2.9979 \cdot 10^{8}$ m/s (roughly a foot per nanosecond)
Planck constant: $h = 6.626 \cdot 10^{-34}$ J s; $\hbar = h / 2\pi = 1.0546 \cdot 10^{-34}$ J s
Fundamental charge unit: $e = 1.6022 \cdot 10^{-19}$ C
Coulomb's Law constant: $1/ 4\pi\varepsilon_0 = 8.988 \cdot 10^{9}$ Nm^2/C^{2s}
Gravitational constant: $G = 6.674 \cdot 10^{-11}$ Nm^2/kg^2
Avogadro constant: $N_A = 6.022 \cdot 10^{23}$ particles per mol; 1 mol = A gram (A = molecular mass)
Boltzmann constant: $k = 1.38 \cdot 10^{-23}$ J/K = $8.617 \cdot 10^{-5}$ eV/K; $R = N_A \cdot k = 8.314$ J/K/mol
Stefan-Boltzmann constant: $\sigma = 5.67 \cdot 10^{-8}$ W/m^{2}K^4
Thompson cross section: $\sigma_e = 6.65 \cdot 10^{-29}\ \text{m}^2$
Electron mass: $m_e = 9.109 \cdot 10^{-31}$ kg
Hydrogen atom (^{1}H) mass: $m_H = 1.6735 \cdot 10^{-27}$ kg ($A = 1.0078$)
Helium atom (^{4}He) mass: $m_{^4He} = 6.6465 \cdot 10^{-27}$ kg ($A = 4.0026$)
Hubble constant: $H_o = 68$ km/s / Mpc
$M_{Earth} = 5.97 \cdot 10^{24}$ kg ; $R_{Earth} = 6.371 \cdot 10^{6}$ m
$M_{sun} = 1.989 \cdot 10^{30}$ kg ; $R_{sun} = 6.955 \cdot 10^{8}$ m, $\Delta_{earth\text{-}sun}$ = 1 A.U. = $1.496 \cdot 10^{11}$ m
$L_{sun} = 3.84 \cdot 10^{26}$ W (corresponds to black-body effective temperature $T = 5777$ K; $M = 4.83$)

Useful conversions:

1 A.U. = $1.496 \cdot 10^{11}$ m; 1 parsec = 1 pc = 206,265 A.U. = $3.086 \cdot 10^{16}$ m = 3.262 light years
Absolute magnitude: $M = m$ at 10 pc distance; $M_2 = M_1 + 2.5 \cdot \log_{10}(L_1/L_2)$
L = luminosity = light energy emitted/second; $M = 71.29 - 2.5\lg(L/\text{Watt})$
Apparent magnitude: $m = M + 5 \cdot \log_{10}(d/10\text{ pc})$; $m_2 = m_1 + 2.5 \cdot \log_{10}(F_1/F_2)$
F = brightness = light absorbed/second/m^2; $F = \dfrac{L}{4\pi d^2}$; $m = 66.29 - 2.5\lg\left(\dfrac{L}{1\text{W}}\right) + 5\lg\left(\dfrac{d}{1\text{ pc}}\right)$

1 fm (= 1 "Fermi") = 10^{-15} m, 1 nm = 10^{-9} m = 10 Å; 1 PHz = 10^{15} Hz

1 eV = $e \cdot 1$V = $1.602 \cdot 10^{-19}$ J (Energy of elementary charge after 1 V potential difference)

1 keV = 1000 eV, 1 MeV= 10^6 eV, GeV = 10^9 eV, 1 TeV = 10^{12} eV

New unit of mass m: 1 eV/c^2 = mass equivalent of 1 eV (Relativity!) = $1.78 \cdot 10^{-36}$ kg

Momentum p: 1 eV/c = $5.34 \cdot 10^{-28}$ kg m/s; p in eV/c = mass in eV/c^2 times velocity in units of c

Planck contant: $\hbar c$ = 197.33 eV nm (1 nm = 10^{-9} m); $\hbar = 6.582 \cdot 10^{-16}$ eVs = 0.658 eV/ PHz

Fine-structure constant: $\alpha = e^2 / 4\pi\varepsilon_0 \hbar c = 1/137.036$

Electron mass: m_e = 510,999 eV/$c^2 \approx$ 0.511 MeV/c^2; Muon mass: m_μ = 105.658 MeV/$c^2 \approx 207 \cdot m_e$

Muon mass: m_μ = 105.658 MeV/$c^2 \approx 207 \cdot m_e$

Proton mass: m_p = 938.272 MeV/$c^2 \approx 1836 \cdot m_e$; Neutron mass: m_n = 939.565 MeV/$c^2 \approx 1839 \cdot m_e$

Neutron mass: m_n = 939.565 MeV/$c^2 \approx 1839 \cdot m_e$

Atomic mass unit (1/12 of the mass of a ^{12}C atom, $\Leftrightarrow A \equiv 1$): u = 931.494 MeV/$c^2 \approx 1823 \cdot m_e$

Rydberg energy: Ry = $m_e c^2 \alpha^2 / 2$ = 13.606 eV

Bohr Radius: $a_0 = \hbar c / (m_e c^2 \alpha)$ = 0.0529 nm (roughly ½ Å).

Planck Length: $l_p = \sqrt{G\hbar / c^3} = 1.616 \cdot 10^{-35}\,\text{m}$. Planck Energy: $E_P = \hbar c / l_P = 1957\,\text{MJ} = 1.22 \cdot 10^{28}\,\text{eV}$

Planck mass: $m_P = E_p/c^2$ = 22 μg. Planck time: $t_P = l_P/c = 5.39 \cdot 10^{-44}$ s.

Special Relativity:

4-vector: Event $(ct, x, y, z) = (ct, \vec{r})$ =: $(x^0, x^1, x^2, x^3) = x^\mu$, $\mu = 0 \ldots 3$

For an inertial system S' moving along the x-axis of S with constant velocity $v < c$, and with all axes aligned and the same origin (x^μ = (0,0,0,0) $\Leftrightarrow$ x'^μ = (0,0,0,0)):

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}; \; x' = \gamma\left(x - \frac{v}{c} ct\right); \; ct' = \gamma\left(ct - \frac{v}{c} x\right); \; y = y'; \; z = z'$$

Clocks in S' appear to S as if they were going slow by factor $1/\gamma$, and vice versa.

Length of object at rest in S' appears contracted by factor $1/\gamma$ in S.

Velocity addition: $$\frac{u_x}{c} = \frac{\frac{u_x'}{c} + \frac{v}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}; \; \frac{u_y}{c} = \frac{\frac{1}{\gamma}\frac{u_y'}{c}}{1 + \frac{u_x'}{c}\frac{v}{c}}.$$

Doppler shift: $\frac{\lambda_{obs}}{\lambda_{emitted}} = (z + 1) = \frac{1 + v_\parallel / c}{\sqrt{1 - v^2 / c^2}}$ (v is the **relative** velocity between emitter and observer and $v_\parallel$ is its component along the line of sight; $z > 0$ is redshift, $z < 0$ is blueshift)

Invariant interval between two events (points in 4-dim. space-time) separated by $\Delta x^\mu = \left(\Delta ct, \Delta\vec{r}\right)$:

$$(\Delta s)^2 = (\Delta ct)^2 - (\Delta\vec{r})^2 = \sum_{\mu,\nu=0\ldots3} \Delta x^\mu g_{\mu\nu} \Delta x^\nu = \begin{pmatrix} \Delta ct & \Delta x & \Delta y & \Delta z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \Delta ct \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

The 4x4 matrix g is called the "metric" - it helps measure distances in terms of coordinates.
The invariant interval has the **same** magnitude in all inertial systems!
Positive $(\Delta s)^2$: "time-like separation" => elapsed "eigen" (proper) time $\Delta\tau = \frac{1}{c}\sqrt{(\Delta s)^2}$ in an inertial system that travels from the start point (event) to the end point (event) of the interval. Negative $(\Delta s)^2$: "space-like separation" (too far for any causal connection between the events).
$(\Delta s)^2 = 0$: "light-like separation"; a light ray could travel from one event to the other.

Four-momentum: $P^\mu = \left(E/c, P_x, P_y, P_z\right) = (\Gamma mc, \Gamma m\vec{u})$; $\Gamma = \dfrac{1}{\sqrt{1-\vec{u}^2/c^2}}$. u = velocity,
$E = P^0 \cdot c$ is total energy of object = sum of rest mass energy ($E_{rest} = mc^2$) plus kinetic energy $T_{kin} = (\Gamma-1)*mc^2$ ($\approx {}^m/_2\, u^2$ **only** if $u \ll c$). Sum of all momenta is conserved in collisions, separately for each component. Transformation of P^μ to coordinate system S' is analog to x^μ (see above).

Invariant Interval: $\left(P^0\right)^2 - \vec{P}^2 = \left(\frac{E}{c}\right)^2 - P_x^2 - P_y^2 - P_z^2 = m^2c^2 \Rightarrow E = c\sqrt{m^2c^2 + \vec{P}^2}$; $\frac{\vec{u}}{c} = \frac{\vec{P}c}{E}$.

Objects with no rest mass (e.g., photons): always $u = c$, $E = |P|c$.

Gravity:

Newtonian Gravity:

Relationship between distance a of two masses M and m and orbital period: $\omega^2 = \dfrac{G(M+m)}{a^3}$

Gravitational potential (=potential energy per unit mass) at a distance r from a spherical mass M:
$\Phi_{grav} = \dfrac{V_{grav}}{m} = -\dfrac{GM}{r}$. Gravitational potential energy of uniform sphere of mass M and radius R:
$V_{grav} = -\dfrac{3}{5}\dfrac{GM^2}{R}$. Virital Theorem: $T_{kin} = -E_{tot} = \frac{1}{2}\left|V_{grav}\right|$ for a system of masses in stable orbits.

General Relativity:

Freely falling reference frames are the new **local** inertial coordinate systems, where photons move with the speed of light in straight lines and a resting object stays at rest in the absence of external forces (**other** than gravity). Objects under the influence of only gravity (no other force) follow "geodesics" (best possible rendition of a "straight line") which maximize the proper time elapsed between 2 points (analog to ordinary straight lines that correspond to the shortest distance between 2 points).

Time Dilation: The local time t_{local} near a massive object elapses more slowly than time t_∞ in the coordinate system of a fixed observer far away: $\Delta t_{local} = \sqrt{1+\frac{2\Phi_{grav}}{c^2}}\Delta t_\infty$; $\Phi_{grav} = \frac{V_{pot}^{grav}}{m}\left(= -\frac{GM}{r}\right)$

(last expression is for spherical mass M)

Schwarzschild radius: $R_s = \frac{2GM}{c^2} \Rightarrow \quad \Delta t_{local} = \sqrt{1-\frac{R_s}{r}}\Delta t_\infty$. Local elapsed time near Schwarzschild radius (= event horizon) becomes zero for any finite remote elapsed time -> all motion appears to come to a standstill as seen from far away (∞ slow "ageing"). => **Schwarzschild metric:**

$$(\Delta s)^2 = \sum_{\mu,\nu=0...3} \Delta x^\mu g_{\mu\nu} \Delta x^\nu = \begin{pmatrix} \Delta ct & \Delta r & \Delta\theta & \Delta\varphi \end{pmatrix} \begin{pmatrix} 1-R_S/r & 0 & 0 & 0 \\ 0 & -1/\left(1-R_S/r\right) & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix} \begin{pmatrix} \Delta ct \\ \Delta r \\ \Delta\theta \\ \Delta\varphi \end{pmatrix}$$

Proper time $\Delta\tau = \frac{1}{c}\sqrt{(\Delta s)^2}$ near spherical mass M: $d\tau = \left(\left[1-\frac{R_s}{r}\right]dt_\infty^2 - \left[1-\frac{R_s}{r}\right]^{-1}\frac{dr^2}{c^2} - \frac{r^2}{c^2}d\theta^2 - \frac{r^2\sin^2\theta}{c^2}d\varphi^2\right)^{1/2}$

Curvature: No **global** inertial coordinate systems are possible in general, since the acceleration of gravity ("free fall") has different magnitudes and different directions at different points in space. Consequence: "straight" lines (= geodesics) become curved (both in real space –> bending of light around massive objects, and in space-time –> different rates of falling at different radial distance); parallel straight lines (light rays) can converge in a single point (Gravitational lensing) etc. => space-time itself is curved! **Consequences**: Light rays bending near massive objects (stars etc.), time-dilation and red-shift of light emitted near massive objects, event horizons,…

Universe at large:

Co-moving coordinate system: $\vec{r}_c = const.$ for a point (object) locally at rest relative to "Hubble flow"; true distance from origin $D = a(t)r_c$. Scale factor $a(t)$ = Radius of curvature in a curved universe (otherwise arbitrary; r_c is meant to be dimensionless). Universal time t (same everywhere; defined through Hubble parameter $H(t)$ – see below). t_0 = present time:

Hubble law: $v_r = \frac{dD}{dt} = \dot{a}(t)r_c = \frac{\dot{a}(t)}{a(t)}D(t) =: H(t)D \Rightarrow \quad H(t) = \frac{\dot{a}(t)}{a(t)}$ (Hubble parameter). At present:

$H_0 = H(t_0) = \frac{68-70 \text{ km/s}}{1 \text{ Mpc}} \approx \frac{1}{14\cdot 10^9\,\text{yr}}$. Speed of light in co-moving coordinates: $\frac{dr_c}{dt} = \frac{c}{a(t)}$

Redshift for light emitted at t and received at t_0: $z = \frac{a(t_0)}{a(t)} - 1$. Invariant distance of object at time of emission: $r_c(em.) = \int_{t_e}^{t_0} \frac{c}{a(t)}dt \Rightarrow D(em.,t_e) = a(t_e)r_c(em.); D(em.,t_0) = a(t_0)r_c(em.)$.

General (Walker-Robertson) metric: $ds^2 = dt^2 - a^2(t)\left[dr_c^2 + S_K^2(r_c)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$.

Critical density: $\rho_c(t) = \frac{3H^2(t)}{8\pi G}$.

Today: $\rho_c(t_0) = \frac{3H_0^2}{8\pi G} \approx 10^{-26}$ kg/m^3 ≈ 6 protons/m^3 ≈ $9 \cdot 10^{-10}$ J/m^3

Closed Universe (positive curvature): $\rho_{tot} = \rho_M + \rho_R + \rho_\Lambda > \rho_c \Rightarrow K = 1, S_K(r_c) = \sin(r_c)$.

Flat Universe (no curvature): $\rho_{tot} = \rho_c \Rightarrow K = 0, S_K(r_c) = r_c$.

Open Universe (negative curvature): $\rho_{tot} < \rho_c \Rightarrow K = -1, S_K(r_c) = \sinh(r_c)$.

Evolution:

$$H^2(t) = \frac{\dot{a}^2(t)}{a^2(t)} = \frac{8\pi}{3} G\rho_{tot}(t) - \frac{Kc^2}{a^2(t)} = H_0^2\left(\frac{\rho_{tot}(t)}{\rho_c(t_0)} - \frac{Kc^2}{H_0^2 a^2(t)}\right) = H_0^2\left(\Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) + \Omega_K(t)\right)$$

$$\Rightarrow \dot{a}(t) = a(t) H_0 \sqrt{\Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) + \Omega_K(t)} \Rightarrow \frac{da}{a} = \sqrt{\Omega_M(t) + \Omega_R(t) + \Omega_\Lambda(t) + \Omega_K(t)}\; H_0 dt$$

with the following ingredients:

1) Matter (both baryonic and dark matter), non-relativistic, density due to mass:

$\Omega_M(t) = \frac{\rho_M(t)}{\rho_c(t_0)} = \frac{\rho_M(t_0)}{\rho_c(t_0)} \frac{a_0^3}{a^3(t)} = \Omega_M^0 \frac{a_0^3}{a^3}$. (Note the “mixed defintion” where ρ_c is always taken at today’s value). Today: $\Omega_M^0 \approx 0.3$, roughly 26% dark matter and 4% baryons.

2) Radiation (any relativistic particles, including photons, neutrinos in the early Universe and ultra-hot matter): $\Omega_R(t) = \frac{\rho_R(t)}{\rho_c(t_0)} = \frac{\varepsilon(t)/c^2}{\rho_c(t_0)} = \frac{\rho_R(t_0)}{\rho_c(t_0)} \frac{a_0^4}{a^4(t)} = \Omega_R^0 \frac{a_0^4}{a^4}$.

$\varepsilon(t)$ = energy density $\propto T^4; T(t) = T(t_0)\frac{a_0}{a(t)}$. Today: $\Omega_R^0 = 8.24 \cdot 10^{-5}$ (mostly due to 2.7 K CMB)

3) Dark energy (cosmological constant Λ): $\Omega_\Lambda(t) = \frac{\rho_\Lambda(t)}{\rho_c(t_0)} = \frac{\rho_\Lambda(t_0)}{\rho_c(t_0)}$ (const.) $= \Omega_\Lambda^0$. Today, $\Omega_\Lambda^0 \approx 0.7$.

4) Curvature: $\Omega_K(t) = -\frac{Kc^2}{H_0^2 a^2(t)} = \Omega_K^0 \frac{a_0^2}{a^2}$. Note: By definition $\Omega_K^0 = 1 - \Omega_M^0 - \Omega_R^0 - \Omega_\Lambda^0$. In principle, can be negative (open Universe) or positive (closed Universe). Today’s value unknown but very close to 0 (within 2%). Must have been extremely close to 0 in the early Universe (Inflation predicts 0).

General behavior of scale factor for different scenarios (dominance of one of the Ω terms):

Matter dominated Universe: $a(t) = a_0\left(1 + \tfrac{3}{2} H_0 t\right)^{2/3}$ Radiation dominance: $a(t) = a_0\left(1 + 2H_0 t\right)^{1/2}$

Dark energy/inflation dominance: $a(t) = a_0 e^{H_0 t}$. (Negative) curvature dominance: $a(t) = a_0 + ct$

Quantum Mechanics:

Formal/abstract (*for experts only*): All knowledge about a system is encoded in state vector $|\psi\rangle$. State vectors can be linearly combined, and we can define a scalar product $\langle\psi'|\psi\rangle$ (complex number). By convention all state vectors are normalized to 1: $\langle\psi|\psi\rangle = 1$.
Observables O are represented by operators Ω with eigenvectors $|\varphi_i\rangle$ and eigenvalues ω_i (real numbers): $\Omega|\varphi_i\rangle = \omega_i|\varphi_i\rangle$. Any measurement of O must give one of these eigenvalues as result. After we measure ω_i, the system will be in the state described by vector $|\varphi_i\rangle$ ("collapse of the state"). The probability to measure this particular eigenvalue is given by $\Pr(\omega_i) = \left|\langle\varphi_i|\psi\rangle\right|^2$. The average (expectation value) for the observable over many independent trials is $\langle O\rangle = \langle\psi|\Omega|\psi\rangle$ with a standard deviation $\Delta O = \sqrt{\langle O^2\rangle - \langle O\rangle^2}$.

Heisenberg's uncertainty principle: Position x and momentum p cannot be predicted with arbitrary precision simultaneously; $\Delta x \Delta p \geq \hbar/2$.

Time evolution (Schrödinger Equation): $|\psi\rangle(t)$; $\frac{\partial}{\partial t}|\psi\rangle(t) = \frac{1}{i\hbar}\mathbf{H}|\psi\rangle(t)$ where **H** is the Hamiltonian operator that represents total mechanical energy (kinetic and potential). Eigenstates of **H**: $\mathbf{H}|\varphi_E\rangle = E|\varphi_E\rangle$; $|\varphi_E\rangle(t) = |\varphi_E\rangle e^{-iEt/\hbar}$ represent bound (stationary) states.

CONSEQUENCES:

1) In general, only probabilistic predictions can be made about measurements of observables
2) Quantization of light (in form of photons) and of energy levels in atoms, nuclei, molecules,…

Hydrogen-like atoms:

(Nucleus of mass m_2 and charge Ze, bound particle of mass m_1 and charge $-e$)

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Z\alpha\hbar c}{r}$$

Strictly speaking, mass must be replaced by "reduced mass" of 2-body system with masses m_1 and m_2: $\mu = \frac{m_1 m_2}{m_1 + m_2} \approx m_1$ if $m_1 << m_2$

Energy Eigenvalues = possible energy levels of "stationary bound states":

$E_n = -\frac{\mu}{m_e}\frac{Z^2}{n^2}Ry \approx -\frac{1}{n^2}Ry$ for hydrogen atom (n = 1, 2, … ; $Ry = m_e c^2 \alpha^2/2$ = 13.6 eV). Degenerate in ℓ and m; ℓ =0, 1,…, n-1, $m_\ell = -\ell \ldots +\ell$; also degenerate in electron spin $m_s = \pm 1/2$ => total degeneracy $g_n = 2n^2$.

Characteristic radius: $a = \frac{m_e}{\mu_r Z}a_0$ $\quad a_0 = \hbar c / (m_e c^2 \alpha) = 0.53$ Å.

Light emitted or absorbed in transition with energy difference $\Delta E = E_{\text{init}} - E_{\text{final}}$:
$f = \Delta E/h$, $\lambda = hc/\Delta E = 2\pi\hbar c/\Delta E$ (Photon energy $E_\gamma = hf$ and momentum $p_\gamma = h/\lambda$)

Pauli principle:

No two identical Fermions (spin-1/2, 3/2, … particles) can be in the same exact quantum state.

Consequence: Only up to two spin-1/2 particles (one with spin "up", one with spin "down") can fit in a "phase-space volume" (= ordinary volume V times momentum-space volume $\frac{4\pi}{3}p_f^3$) of size h^3 (Planck's constant cubed). Therefore, a "Fermi gas" (also called a "degenerate gas") of spin-1/2 particles has to occupy available momentum states up to a maximum of the Fermi momentum $p_f = \hbar\left(3\pi^2\right)^{1/3} n^{1/3}$; $n = \frac{N_{tot}}{V}$ where N_{tot} is the total number of Fermions. As a consequence, the minimum total kinetic energy for a Fermi gas with N_{tot} identical fermions in a sphere of radius R is

$$E_{tot}^{kin} = \begin{cases} \frac{3}{5}N_{tot}\frac{p_f^2}{2m} = \frac{3\hbar^2}{10m}N_{tot}\left(3\pi^2\right)^{2/3}\left(\frac{N_{tot}}{V}\right)^{2/3} = \frac{3\hbar^2\left(\frac{9\pi}{4}\right)^{2/3}}{10m}\frac{N_{tot}^{5/3}}{R^2}\text{; non-relativistic} \\ \frac{3}{4}N_{tot}cp_f = \frac{3}{4}\hbar cN_{tot}\left(3\pi^2\right)^{1/3}\left(\frac{N_{tot}}{V}\right)^{1/3} = \frac{3\hbar c\left(\frac{9\pi}{4}\right)^{1/3}}{4}\frac{N_{tot}^{4/3}}{R}\text{; ultra-relativistic} \end{cases}$$

Astrophysics:

For a white dwarf of mass M, equilibrium between gravity and "Fermi pressure" is reached when $R = \frac{\hbar^2 N_{tot}^{5/3}}{m_e G M^2}\left(\frac{9\pi}{4}\right)^{2/3}$ with $N_{tot} = \frac{M}{1.008\text{ g}}\frac{N_A}{2}$. (**careful**: 1 g = 0.001 kg!)

For a neutron star, replace m_e with m_n and N_{tot} with $N_{tot} = \frac{M}{1.009\text{ g}}N_A$.

Chandrasekar limit: White dwarfs become unstable beyond $M = 1.4\ M_{\text{sun}}$; neutron stars become unstable beyond 2 M_{sun} (both due to relativistic effects: the kinetic energy goes up only like $1/R$ instead of $1/R^2$ as the Fermi momentum approaches the mass of the fermion in magnitude).

Nuclear Physics

Mass-energy of an atom: (*Z* protons, *N* neutrons, $A = Z+N$):

$M_A c^2 = Z\, M_p c^2 + N\, M_n c^2 + Z\, m_e c^2 - BE$ (Binding energy)

typical binding energies $BE = 7\text{-}8$ MeV$\cdot A$ with a maximum of BE/A for nuclei around iron (A=56). Light nuclei have significantly lower *BE* per nucleon; beyond iron, the *BE* per nucleon decreases slowly with *A* (due to Coulomb repulsion).

Energy liberated during a nuclear fusion reaction 1 + 2 -> 3: $\Delta E = M_1c^2 + M_2c^2 - M_3c^2$

Energy liberated during a nuclear decay 1 -> 2 + 3: $\Delta E = M_1c^2 - M_2c^2 - M_3c^2$

Density: roughly constant $\rho = 0.16$ Nucleons / fm^3 = 2×10^{17} kg/m^3

Radioactive nuclei:

alpha-decay: $(Z,A) \rightarrow (Z\text{-}2, A\text{-}2) + {}^4\text{He}$ + energy

beta-plus decay: $(Z,A) \rightarrow (Z\text{-}1, A) + e^+ + \nu_e$

beta-minus decay: $(Z,A) \rightarrow (Z+1, A) + e^- + \bar{\nu}_e$

Decay probability in time Δt: $\Delta\text{Pr}(\Delta t) = \Delta t/\tau$ (τ = lifetime = $T_{1/2}$ / ln 2)

Number of undecayed nuclei at time *t* (starting with N_0): $N(t) = N_0\, e^{-t/\tau}$

Particle Physics

Fundamental Fermions (spin-1/2 particles obeying Pauli Exclusion Principle):
quarks (up, down, charm, strange, top, bottom) and leptons (electron, muon, tau, electron-neutrino, muon-neutrino, tau-neutrino) and their antiparticles:

Name	Symbol	Mass (MeV/c^2)*	*J*	*B*	*Q* (e)
Up	u	$2.3^{+0.7}_{-0.5}$	½	+⅓	+⅔
Down	d	$4.8^{+0.5}_{-0.3}$	½	+⅓	−⅓
Charm	c	1275 ±25	½	+⅓	+⅔
Strange	s	95 ±5	½	+⅓	−⅓
Top	t	173 210 ±510 ± 710	½	+⅓	+⅔
Bottom	b	4180 ±30	½	+⅓	−⅓

Particle/antiparticle name	Symbol	Q (e)
Electron / Positron[18]	e^- / e^+	−1 / +1
Muon / Antimuon[19]	μ^- / μ^+	−1 / +1
Tau / Antitau[21]	τ^- / τ^+	−1 / +1
Electron neutrino / Electron antineutrino[34]	ν_e / $\bar{\nu}_e$	0
Muon neutrino / Muon antineutrino[34]	ν_μ / $\bar{\nu}_\mu$	0
Tau neutrino / Tau antineutrino[34]	ν_τ / $\bar{\nu}_\tau$	0

Force-mediating Gauge Bosons (spin-1 particles obeying Bose-Einstein statistics):
Photon γ (electromagnetic interaction), W^+, W^-, Z^0 (weak interaction), gluons (strong interaction) [graviton (gravity) only conjectured]. All except weak interaction bosons are massless; the latter gain mass (80-91 GeV/c^2) through interaction with the Higgs field.

Thermal/Statistical Physics

Boltzmann Distribution: number $n(E)$ of atoms (molecules, …) out of an ensemble with a total of N atoms (…) with given energy E in a system with absolute temperature T (in K).

Discrete energy levels E_i (*e.g.*, quantum systems) with degeneracy g_i (= number of eigenstates of the Hamiltonian with energy eigenvalue E_i):

$$n(E_i) = Cg_i e^{-E_i/kT} = \frac{g_i}{e^{(E_i-\mu)/kT}}; C = e^{\mu/kT} = N / \sum g_i e^{-E_i/kT}$$

(C is a normalization constant; μ is the "chemical potential")

Continuous energy levels E (classical system, e.g. monatomic gas) with state density $g(E)\mathrm{d}E$ (= volume in "phase space" between energy E and energy $E + \mathrm{d}E$):

$$dn(E...E+dE) = C\,g(E)dE\,e^{-E/kT}; C = N / \int g(E)\,dE\,e^{-E/kT}$$

State density for simple monatomic gas:

$$g(E)dE = 4\pi p^2 dp = 4\pi m\sqrt{2mE}\,dE$$

Consequences: Ideal gas law $PV = nRT = nN_A kT$, (n = number of mols; $N = nN_A$); average energy per degree of freedom (dimension of motion) = ½ kT => total kinetic energy of a monatomic gas = 3/2 kT per atom or $E_{tot} = {}^3/_2\, nN_A kT = {}^3/_2\, nRT$

Fermi-Dirac Distribution (for a system of indistinguishable Fermions):

$$n(E_i) = N\frac{g_i}{e^{(E_i-\mu)/kT}+1}$$; μ here is right above the Fermi energy = the highest filled energy level necessary to accommodate all N fermions, where all lower energy levels are filled with as many Fermions as the Pauli principle allows
(= the state of a (degenerate) Fermi gas at close to zero temperature).

Bose-Einstein Distribution (for a system of indistinguishable bosons):

$$n(E_i) = N\frac{g_i}{e^{(E_i-\mu)/kT}-1}$$; μ here is right below the ground state energy (the lowest available energy level). If T goes to zero, all levels but that lowest energy level are empty = Bose-Einstein condensation.

Photon density for black-body radiation: $\frac{dn_\gamma(\lambda\ldots\lambda+d\lambda)}{dV} = \frac{8\pi}{\lambda^4}\frac{d\lambda}{e^{hc/\lambda kT}-1} = 8\pi\frac{f^2}{c^3}\frac{df}{e^{hf/kT}-1}$

Energy density (= energy contained in electromagnetic radiation of wave length λ, per unit volume V) for black-body radiation (i.e., Bose-Einstein Distribution for a photon gas:

$\frac{dE}{V} = 8\pi h\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1} = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$; Energy flux/surface area $\frac{dE}{dAdt} = \frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$

(Planck's Law); Maximum for $\lambda = \frac{hc}{4.9663kT} = \frac{2.9\text{ mm}}{T[K]}$. Total over all wave lengths: σT^4

Propagation of electromagnetic waves

Energy per volume dV in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dV}=u_\lambda d\lambda\text{ ; }u_\lambda\text{ = specific energy density.}$$

Example: black-body $u_\lambda=\frac{dn_\gamma}{d\lambda}\frac{hc}{\lambda}=\frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$

Total (integral over all wave lengths): $dE_{tot}/dV = 4\sigma/c\ T^4$

Maximum intensity per unit wave length interval for λ = 2.9 mm / T [K] (Wien displacement law)

Total power emitted (intensity integrated over all wave lengths) $I = \sigma T^4$ (Stefan-Boltzmann equation)

Total luminosity of spherical black-body of radius R: $L=4\pi R^2\sigma T^4$

Apparent brightness (flux density) at distance d: $F=\frac{L}{4\pi d^2}$

Power emitted per area dA into solid angle $d\Omega$ in wave length interval $d\lambda$:

$$\frac{dE(\lambda\ldots\lambda+d\lambda)}{dt\,dA\,d\Omega}=\cos\theta\cdot I_\lambda(\theta,\varphi)\,d\lambda\text{ ; }I_\lambda\text{ = specific intensity.}$$

Average: $\langle I_\lambda\rangle=\frac{1}{4\pi}\iint I_\lambda(\theta,\varphi)\,d\Omega=\frac{c}{4\pi}u_\lambda$. Ex.: black-body: $\langle I_\lambda\rangle=\frac{2hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$

Power emitted in positive (neg.) z-direction per area dA perpendicular to z and per $d\lambda$:

Radiation flux density $F_\lambda=\int_0^{2\pi}d\varphi\int_{0\,(\pi/2)}^{\pi/2\,(\pi)}\cos\theta\cdot I_\lambda(\theta,\varphi)\sin\theta\,d\theta$

For isotropic specific intensity (for top hemisphere): $\Rightarrow F_\lambda=\pi\langle I_\lambda\rangle$

Black-body radiation: $F_\lambda d\lambda=\frac{2\pi hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}=2\pi hc\frac{f^3}{c^3}\frac{df}{e^{hf/kT}-1}$

Radiation pressure in z-direction: $dP_\lambda^z=\frac{2}{c}d\lambda\int_0^{2\pi}d\varphi\int_{0(\pi/2)}^{\pi/2(\pi)}\cos^2\theta\cdot I_\lambda(\theta,\varphi)\sin\theta\,d\theta$

For isotropic specific intensity in top hemisphere: $\Rightarrow dP_\lambda^z=\frac{4\pi}{3c}\langle I_\lambda\rangle d\lambda=\frac{1}{3}u_\lambda d\lambda\Rightarrow P_{tot}=\frac{1}{3}\frac{E_{tot}}{V}$

Scattering probability: $d\Pr=\frac{N_{atoms}}{V}\sigma ds=\frac{\rho}{m_{Atom}}\sigma ds=\rho\kappa ds$; σ = cross section, ρ = density.

$\kappa=\sigma/m_{Atom}$ = opacity. Mean free path $\ell=1/\kappa\sigma$. $\tau = D/\ell$ = optical depth. Total pathlength = $D\tau$. For ionized star atmospheres (70% H), Rosseland opacity $\bar{\kappa}\approx 0.034\ \text{m}^2/\text{kg}$. For fully ionized hydrogen, $\bar{\kappa}=\frac{\sigma_e}{m_H}=0.042\ \text{m}^2/\text{kg}$ => $L_{\max}=\frac{4\pi GMc}{\bar{\kappa}}=(3.3-3.8)\cdot 10^4\frac{L_{sun}M}{M_{sun}}$ (**Eddington Luminosity Limit**).