Generalized Parton Distributions (GPDs)

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Scattering:
A+B→C$_1$+C$_2$…C$_n$

$$d\sigma = \frac{(2\pi)^4}{4\sqrt{(p_A \bullet p_B)^2 - m_A^2 m_B^2}} \left|\langle f|\hat{T}|i\rangle\right|^2 d^n LIPS$$

$$d^n LIPS = \frac{1}{(2\pi)^{3n}} \frac{d^3\vec{p}_1}{2E_1} \frac{d^3\vec{p}_2}{2E_2} \cdots \frac{d^3\vec{p}_n}{2E_n} \delta^{(4)}\left(p_A + p_B - \sum_{j=1}^{n} p_j\right)$$

- T = Scattering Matrix
- In Non-Relativistic Q.M., T is the solution of the (integral) Lippmann-Schwinger equation:

$$\hat{T} = V + V \frac{1}{E - H_0 + i\varepsilon} \hat{T}$$

Born Approximation

- If the interaction responsible for the scattering is sufficiently weak, we can truncate the expansion: $T \approx V$
- Examples
  - Electromagnetic interactions with large momentum transfer
  - $W^{\pm}$, Z^0 exchange (Weak interactions)
  - pion-nucleon scattering as $p \to 0$
- Counter-examples
  - Photon-atom interactions near a resonance
  - Nucleon-Nucleon scattering.

High Energy Electron Scattering

$$\hat{T} \rightarrow \int d^4 x A_\mu(x) \hat{J}^\mu(x)$$

$$\hat{J}^\mu(x) = \sum_{f=u,d,s,c,t,b} e_f \hat{\bar{\psi}}_f(x) \gamma^\mu \hat{\psi}_f(x)$$

$$\Box A^\mu(x) = j_e^\mu(x) = \bar{u}_e(k',h')\gamma^\mu u_e(k,h) e^{i(k'-k)\cdot x}$$

$$A^\mu(x) = \bar{u}_e(k',h')\gamma^\mu u_e(k,h) \frac{e^{-iq\cdot x}}{-q^2}$$

$$\hat{\psi}_f(x) = \text{Quark field operator of flavor } f$$

$$= \int \frac{d^3 k}{(2\pi)^{3/2}\sqrt{E_k}} \sum_h \left[\hat{b}(k,h) u_f(k,h) e^{-ik\cdot x} + \hat{d}^\dagger(k,h) v_f(k,h) e^{ik\cdot x} \right]$$

Elastic ep Scattering

$$i\mathcal{M} = \left\langle k'P' \right| \int d^4x A_\mu(x) \hat{J}^\mu(x) \left| kP \right\rangle$$

$$= \bar{u}_e(k',h')\gamma^\mu u_e(k,h) \frac{1}{-q^2} \left\langle P' \right| \int d^4x e^{-iq\cdot x} \hat{J}^\mu(x) \left| P \right\rangle$$

$$\left\langle P' \right| \hat{J}^\mu(x) \left| P \right\rangle = \left\langle P' \right| e^{+i\hat{P}\cdot x} \hat{J}^\mu(0) e^{-i\hat{P}\cdot x} \left| P \right\rangle = e^{i(P'-P)\cdot x} \left\langle P' \right| \hat{J}^\mu(0) \left| P \right\rangle$$

$$i\mathcal{M} = \bar{u}_e(k',h')\gamma^\mu u_e(k,h) \frac{1}{-q^2}$$

$$\left\langle P' \right| \sum_{f=u,d,s,c,t,b} e_f \hat{\bar{\psi}}_f(0)\gamma^\mu \hat{\psi}_f(0) \left| P \right\rangle \int d^4x e^{i(P'-P-q)\cdot x}$$

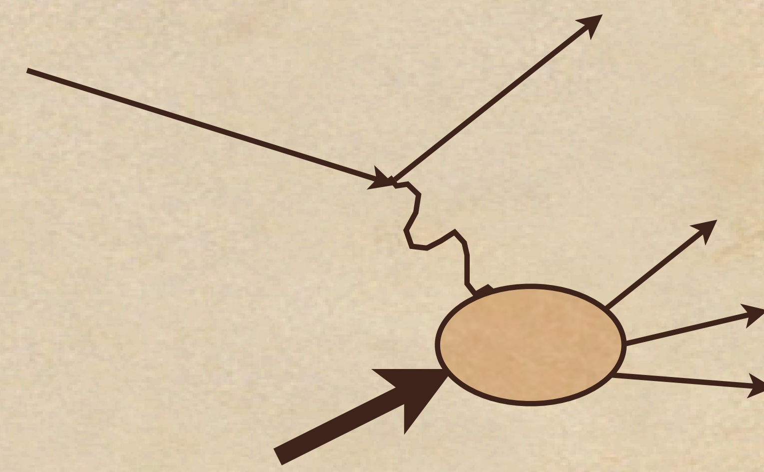
Elastic Form Factors

$$\langle P's'|\sum_{f=u,d,s,c,t,b} e_f \hat{\bar{\psi}}_f(0)\gamma^\mu \hat{\psi}_f(0)|Ps\rangle = \bar{U}(P',s')\left[\gamma^\mu F_1(-q^2) + \frac{[\gamma^\mu,\gamma^\nu]q_\nu}{2M}F_2(-q^2)\right]U(P,s)$$

- This is the most general vector current expressed in proton variables (not quark variables)

Deep Inelastic Scattering

e+p→e'+X



$$i\mathcal{M} = j_\mu \frac{1}{-q^2}\langle X|\int d^4x e^{-iq\cdot x}\hat{J}^\mu(x)|P\rangle$$

$$|i\mathcal{M}|^2 = j_\nu{}^\dagger j_\mu \frac{1}{q^4}\langle P|\int d^4y e^{iq\cdot y}\hat{J}^{o\dagger}(y)|X\rangle\langle X|\int d^4x e^{-iq\cdot x}\hat{J}^\mu(x)|P\rangle$$

$$\sum_X |i\mathcal{M}|^2 = j_\nu^\dagger j_\mu \frac{1}{q^4}\Im m\left[\int d^4y d^4x e^{iq\cdot(y-x)}\langle P|\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\hat{\bar{\psi}}(x)\gamma^\mu\hat{\psi}(x)|P\rangle\right]$$

- As $-q^2\to\infty$, $e^{iq(y-x)}\to 0$ unless $(y-x)^2\to 1/q^2$
- Short distance propagation from x to y is perturbative in QCD (asymptotic freedom)

Light Cone Variables

- Pick a 'privileged' z-axis 'x^3'
- $x^{\pm} = (x^0 + x^3)/2^{1/2}$, $\mathbf{x}_\perp = (x_1, x_2)$
  - $a \bullet b = a^+ b^- + a^- b^+ - \boldsymbol{a}_\perp \bullet \boldsymbol{b}_\perp$
  - $x_\mu x^\mu = 2x^+ x^- - \mathbf{x}_\perp \cdot \mathbf{x}_\perp$
- DIS: Pick $-z$ along $\mathbf{q}$-direction

$$P = \left[P^+, 0_\perp, P^- = \frac{M^2}{2P^+}\right]$$

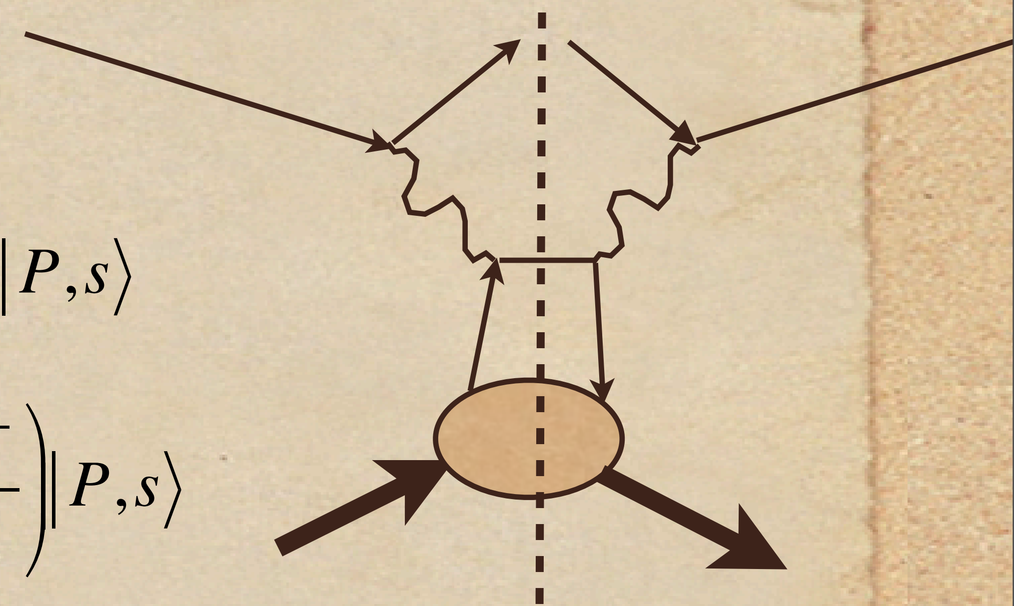
$$q = \left[\frac{-Q^2}{2q^-}, 0_\perp, q^-\right] = \left[-x_B P^+, 0_\perp, q^-\right]$$

$$x_B = \frac{Q^2}{2q \bullet P}$$

DIS Structure Functions

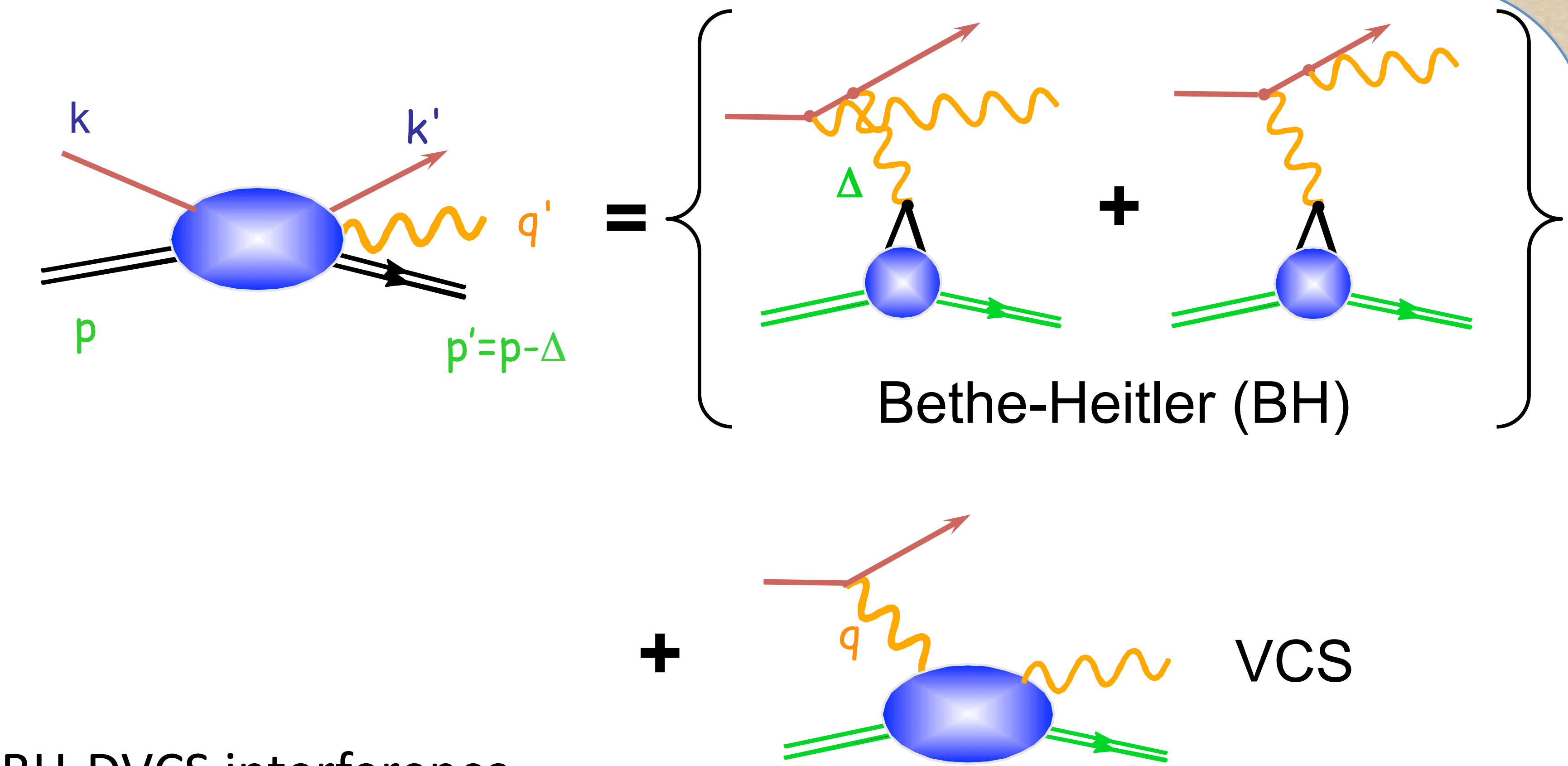
$$F_1(x_B;Q^2) = \int dz^- e^{ix_B P^+ z^-} \sum_{f,s} e_f^2 \langle P,s | \bar{\psi}\left(\frac{z^-}{2}\right) \gamma^+ \psi\left(\frac{-z^-}{2}\right) \| P,s \rangle$$

$$g_1(x_B;Q^2) = \int dz^- e^{ix_B P^+ z^-} \sum_{f,s} e_f^2 \langle P,s | \bar{\psi}\left(\frac{z^-}{2}\right) \gamma^+ \gamma_5 \psi\left(\frac{-z^-}{2}\right) \| P,s \rangle$$



- Forward, non-local matrix element
  - Insert gauge link $e^{i\int A dl}$ from $-z^-/2$ to $z^-/2$

DVCS ep→epγ

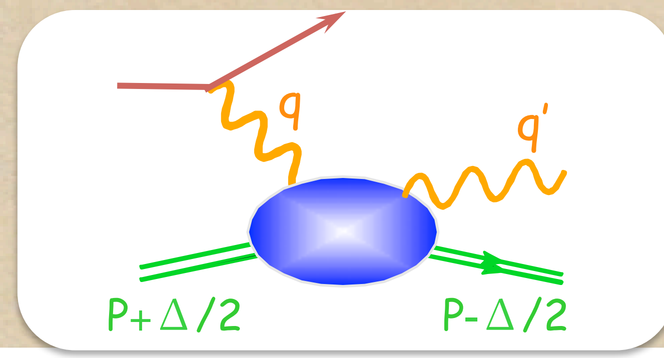


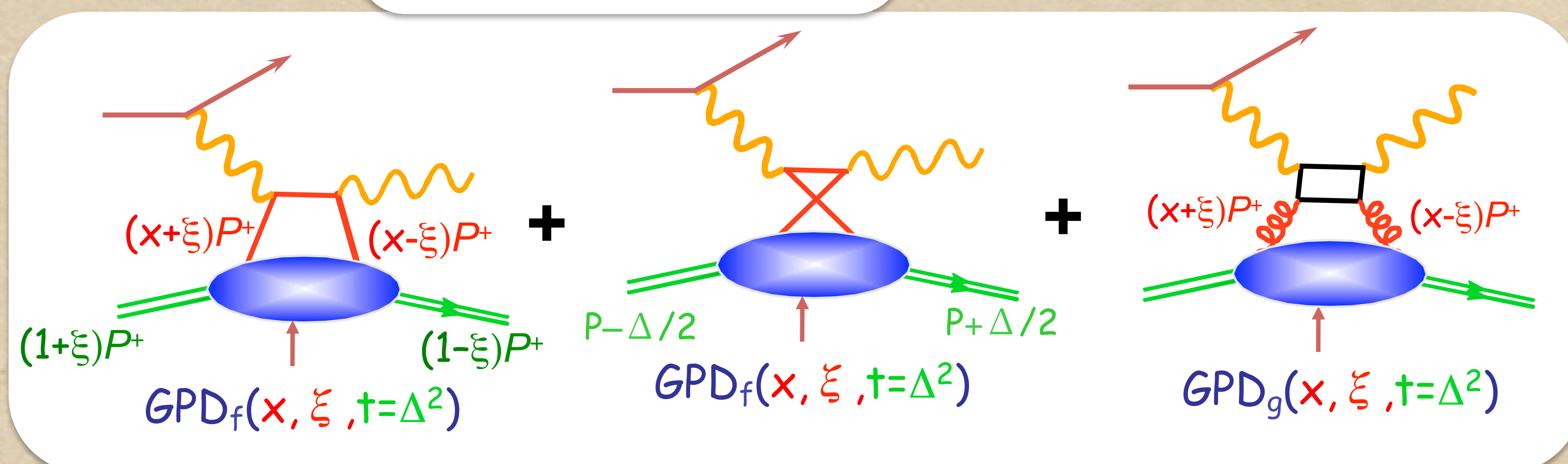
- BH-DVCS interference
  - Access to VCS amplitude, linear in GPDs

- Symmetrized LightCone Coordinates:
  - $P = (p+p')/2=[P^+,0_\perp,P^-]$
  - $P^- = [M^2-\Delta^2]/(2P^+)$

QCD Factorization:

$Q^2 \to \infty$

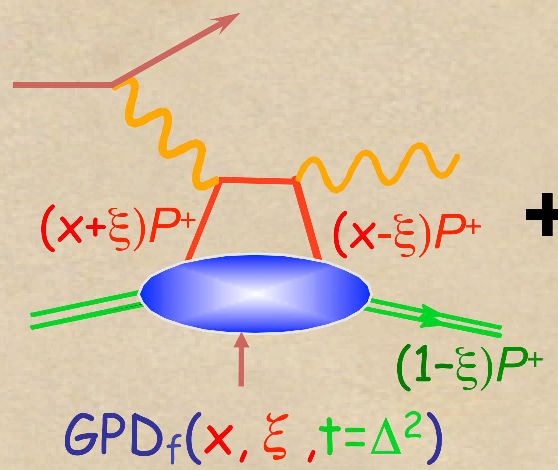




$$\xi = \frac{-(q+q')^2}{2(q+q')\cdot P} \xrightarrow[\Delta^2 << Q^2]{} \frac{x_B}{2-x_B}$$

Generalized Parton Distributions

- Non-local, Off-forward
  `Light-ray' matrix elements



$$\int \frac{dz^- P^+}{2\pi} e^{ixP^+z^-} \langle p',s' | \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+ \psi_f\left(\frac{-z^-}{2}\right) \| p,s\rangle$$

$$= \bar{U}(p',s')\left[H_f(x,\xi,\Delta^2)\gamma^+ + E_f(x,\xi,\Delta^2)\frac{\left[\gamma^+,\gamma^\nu\right]\Delta_\nu}{M}\right]U(p,s)$$

$$\int \frac{dz^- P^+}{2\pi} e^{ixP^+z^-} \langle p',s' | \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+ \gamma_5 \psi_f\left(\frac{-z^-}{2}\right) \| p,s\rangle$$

$$= \bar{U}(p',s')\left[\tilde{H}_f(x,\xi,\Delta^2)\gamma^+\gamma_5 + \tilde{E}_f(x,\xi,\Delta^2)\frac{\Delta^+}{2M}\gamma_5\right]U(p,s)$$

Properties of GPDs:
Forward Limit

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P',s' \right| \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+\psi_f\left(\frac{-z^-}{2}\right)\left| P,s \right\rangle \xrightarrow[P'\to P,\; s'=s]{}$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle P,s \right| \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+\psi_f\left(\frac{-z^-}{2}\right)\left| P,s \right\rangle$$

$$= q_f(x)\theta(x) - \bar{q}_f(-x)\theta(-x)$$

- As (q-q') → 0:
  - $H_f(x,\xi,\Delta^2) \to q_f(x)\theta(x) - \bar{q}_f(-x)\theta(-x)$
  - $\tilde{H}_f(x,\xi,\Delta^2) \to \Delta q_f(x)\theta(x) + \Delta\bar{q}_f(-x)\theta(-x)$

Properties of GPDs:
Moments (Integrals over x)

$$\int dx \int \frac{dz^- P^+}{2\pi} e^{ixP^+z^-} \left\langle P',s' \middle| \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+\psi_f\left(\frac{-z^-}{2}\right) \middle| P,s \right\rangle$$

$$= \int \frac{dz^- P^+}{2\pi} \left\langle P',s' \middle| \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+\psi_f\left(\frac{-z^-}{2}\right) \middle| P,s \right\rangle \int dx e^{ixP^+z^-}$$

$$= \int \frac{dz^- P^+}{2\pi} \left\langle P',s' \middle| \bar{\psi}_f\left(\frac{z^-}{2}\right)\gamma^+\psi_f\left(\frac{-z^-}{2}\right) \middle| P,s \right\rangle \frac{2\pi}{P^+}\delta(z^-)$$

$$= \left\langle P',s' \middle| \bar{\psi}_f(0)\gamma^+\psi_f(0) \middle| P,s \right\rangle = \bar{U}(p',s')\left[F_{1f}(-\Delta^2)\gamma^+ + F_{2,f}(-\Delta^2)\frac{\left[\gamma^+,\gamma^\nu\right]\Delta_\nu}{M}\right]U(p,s)$$

- First Moments:

$$\int dx H_f(x,\xi,\Delta^2) = F_{1,f}(-\Delta^2)$$

$$\int dx E_f(x,\xi,\Delta^2) = F_{2,f}(-\Delta^2)$$

$$\int dx \tilde{H}_f(x,\xi,\Delta^2) = G_{A,f}(-\Delta^2)$$

$$\int dx \tilde{E}_f(x,\xi,\Delta^2) = G_{P,f}(-\Delta^2)$$

X. Ji Angular Momentum Sum-Rule

$$Lim_{\Delta^2 \to 0} \int x\,dx \left\{ H_f(x,\xi,\Delta^2) + E_f(x,\xi,\Delta^2) \right\} = J_f$$

- Total contribution (orbital Angular momentum plus spin) of quarks of flavor f to proton spin.

Challenges and Opportunities with Generalized Parton Distributions

- DVCS and Deep Virtual Meson Production (DVMP) measure GPDs on $x=\pm\xi$ line.
  - DDVCS and QCD Evolution give limited access to $|x|\neq\xi$
  - Angular momentum sum-rule at constant ξ
    and probability imaging at $\xi=0$ are not directly accessible experimentally.
- GPDs are the Twist-2 signal of DVCS and DVMP. How do we isolate this from higher twist (is Q^2-dependence enough?)
  - Higher twist contains important Quantum correlations!
- How can we determine the appropriate meson Distribution Amplitudes for evaluation of DVMP?: Asymptotic? Flat?
  - HERA data suggests factorization works for $Q^2 \sim 5$ GeV^2,
    but strong finite size corrections persist up to > 10 GeV^2

SPATIAL IMAGING at $\xi=0$ and at $x=\xi$

Charles Hyde

Informal Pre-Town Meeting at JLab

August 13 - 15, 2014

Regge-Inspired Model

- M.Diehl, P. Kroll, *Eur.Phys.J. C* **73** (2013) 2397.

  - $H_f(x,0,\Delta^2) = q_f(x) exp[\Delta^2 B_{1f}(x)]$
    $E_f(x,0,\Delta^2) = e_f(x) exp[\Delta^2 B_{2f}(x)]$

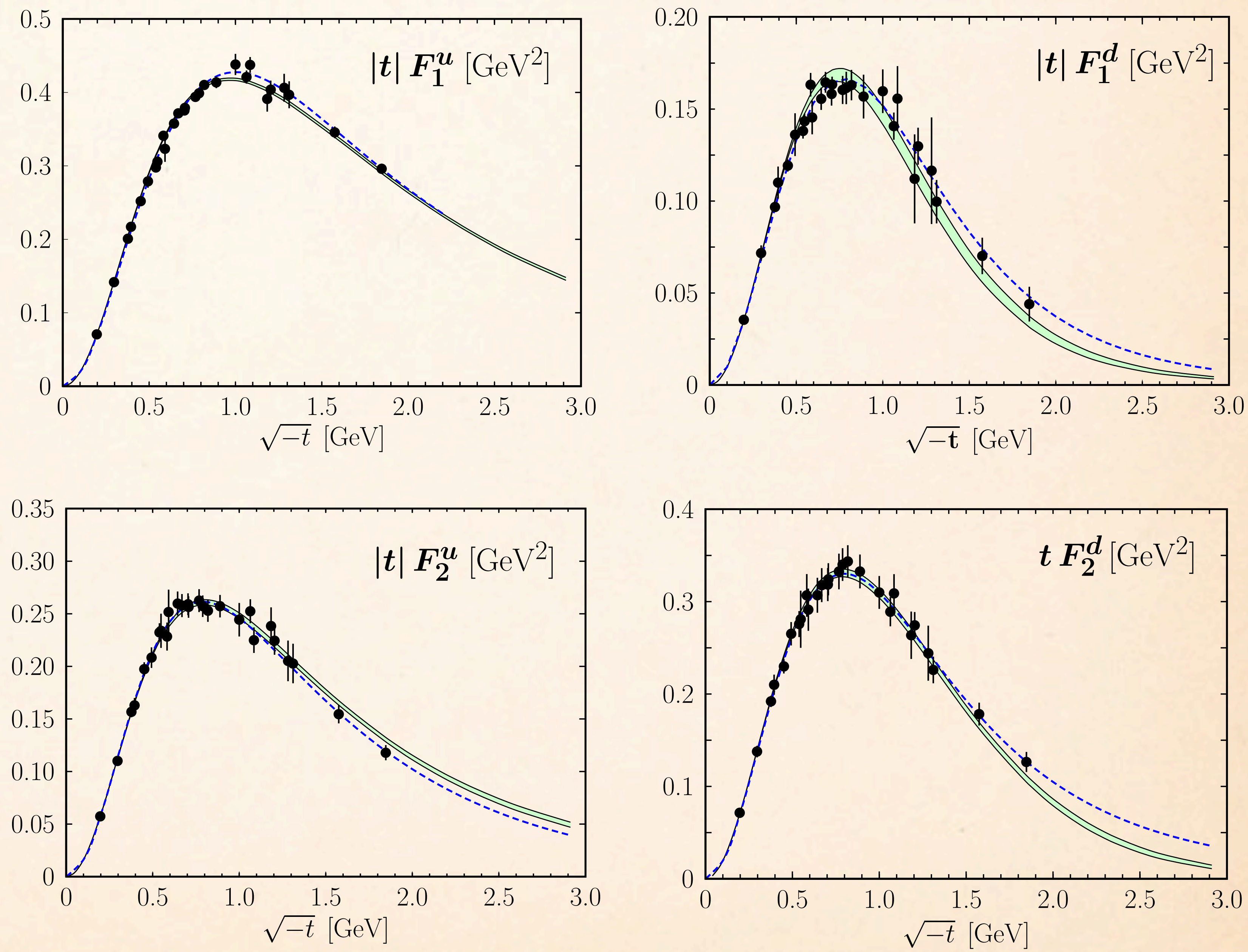
    - $q_f(x)$: *ABM2011*
      $e_f(x) = \kappa_f N_f\, x^{-\alpha f} (1-x)^{-\beta f} (1-\gamma_f\, x^{1/2})$

    - $B_{nf}(x) = \alpha_f' (1-x)^3 log(1/x) + A_{nf}\, x(1-x)^2 + B_{nf} (1-x)^3$

  - Fit:
    $\int dx\, H_f(x,0,\Delta^2) = F_{1f}(-\Delta^2)$
    $\int dx\, E_f(x,0,\Delta^2) = F_{2f}(-\Delta^2)$

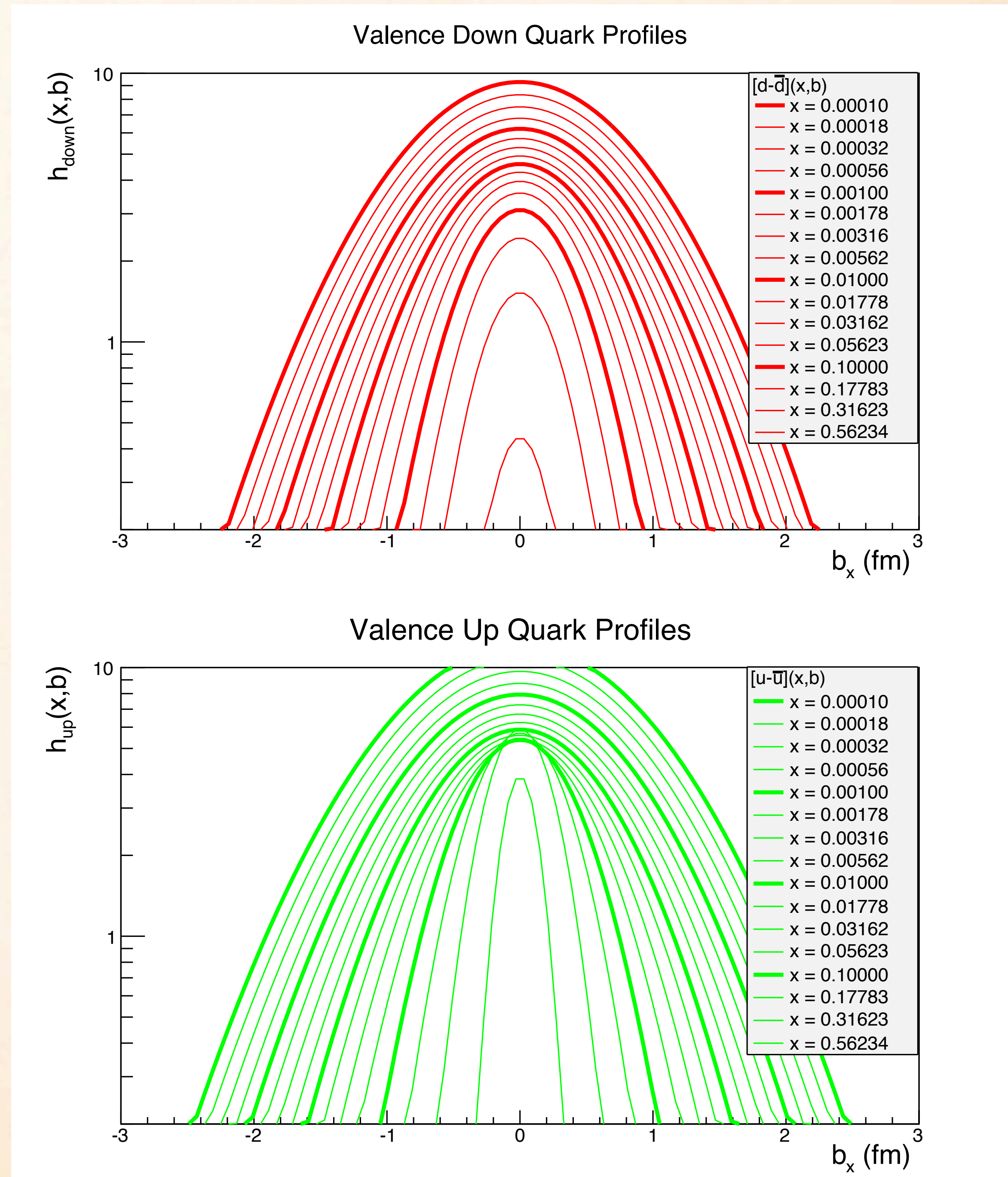
Form Factor Fits

- Non-trivial t-dependence from x-dependent simple Regge slopes
- All the funny little wiggles in $G_{E,M}(-t)$ are resolved into a smooth behavior of the flavor separated $F_{1,2}$



Spatial Densities at ξ=0

- x-dependent t-slope $B(x)$
  - Simple Gaussians in impact parameter space (b_x, b_y)
- Gaussian width strongly x-dependent
  - Negative charge density at center of neutron
- Scale: $\mu^2 = 2$ GeV^2.

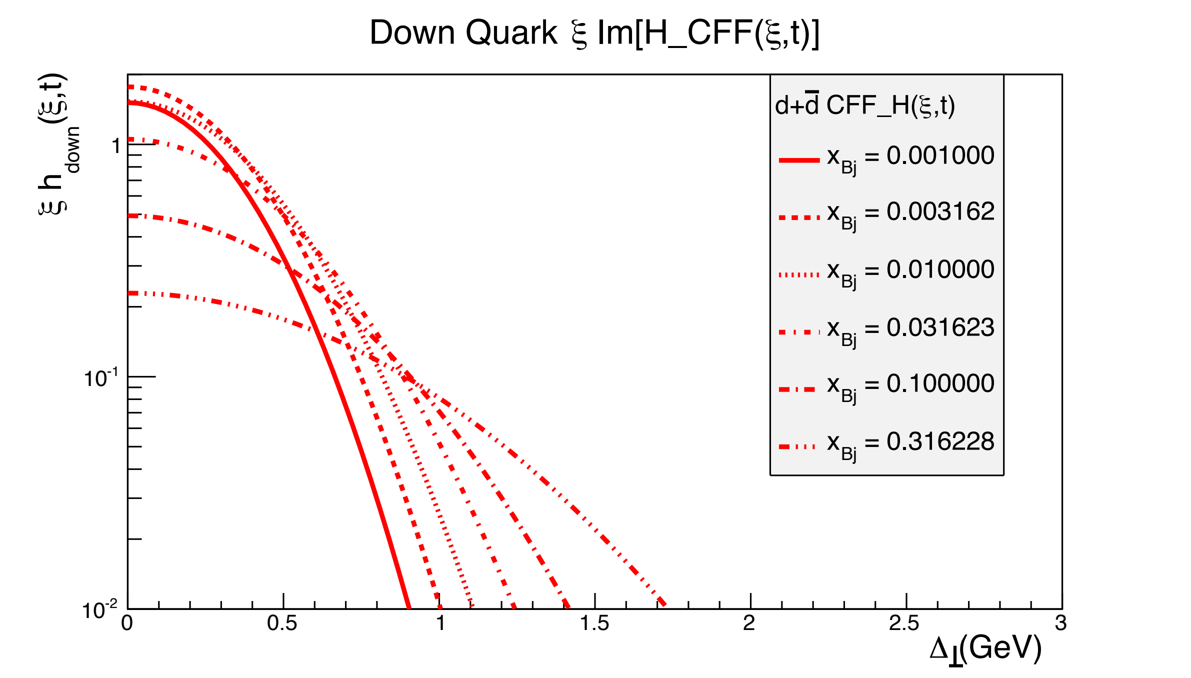


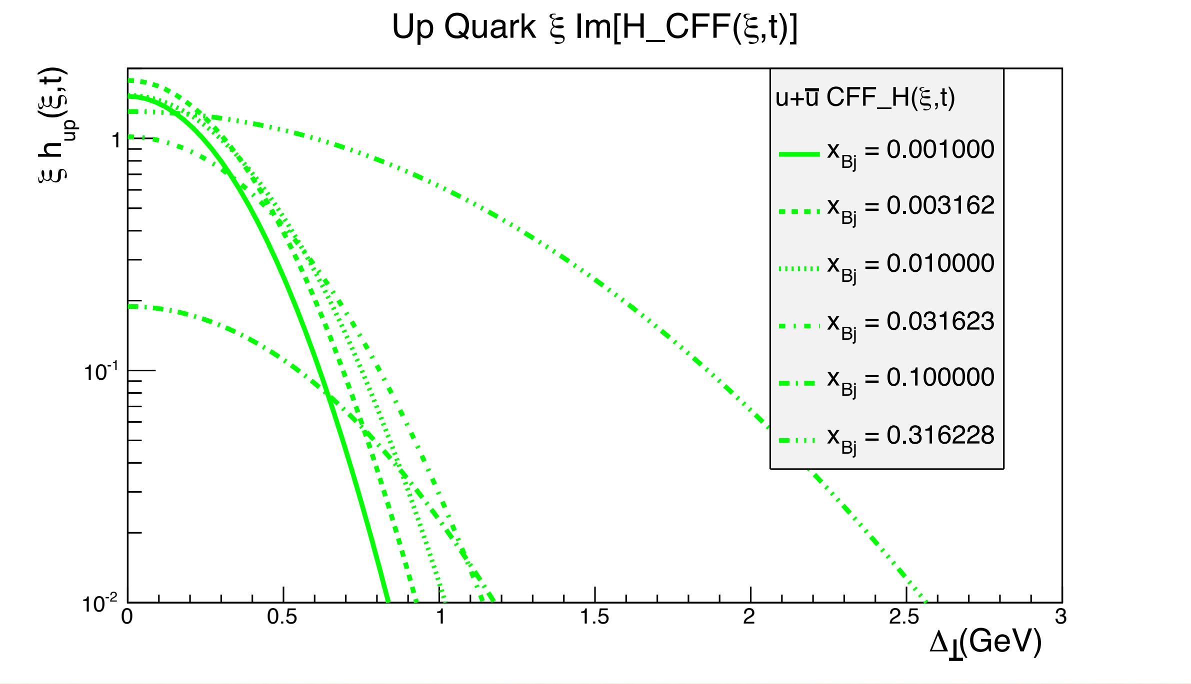
Double-Distribution GPDs at $x=\pm\xi$

- Compton Form Factor: $\xi = x_{Bj}/(2-x_{Bj})$
  $Im[\mathcal{H}_f(\xi,\Delta^2)] = \pi[H_f(\xi,\xi,\Delta^2) - H_f(-\xi,\xi,\Delta^2)]$
- $$\xi\,\mathrm{Im}\left[H_f\left(\xi,\Delta^2\right)\right] = \pi\int_0^{x_{Bj}} d\beta\left[q_f(\beta)+\bar{q}_f(\beta)\right]\left[h_f(\alpha,\beta)\right]_{\alpha=1-\beta/\xi} e^{\Delta^2 B_{1f}(\beta)}$$
- Profile functions $h(\alpha,\beta)$ arbitrary:
  - Use: $h(\alpha,\beta) = N_1 \dfrac{\left[\left(1-|\beta|\right)^2-\alpha^2\right]}{\left(1-|\beta|\right)^3}$
- M. Burkardt, arXiv:*0711.1881* $\qquad \Delta^2 = -\dfrac{4\xi^2M^2+\Delta_\perp^2}{1-\xi^2}$
  $\Delta_\perp$: Fourier Conjugate to $\mathbf{r}_\perp$, the transverse spatial separation between the active parton and the transverse spatial Center-of-Momentum of ***the spectator system***.

Compton Form Factors on the $x=\pm\xi$ line

- Compton Form Factors: $x = \pm\xi$ profiles of GPDs:
- Radial size: strongly ξ-dependent
- Flavor, gluon variation is measureable
  - Intriguing insight into dynamics without sum-rules or extrapolation to $\xi=0$





IMAGING

- In the Photoshop era, you don't have to be a Philosopher or a Surrealist to understand that the image of an object is **not** the object.

- *$[H_f(\xi, \xi, \Delta^2) - H_f(-\xi, \xi, \Delta^2)]$* **is** an image of the proton.
  - It is a non positive-definite quantum transition density, but it still can be interpreted physically.